

Exploring porosity impact on the free vibration of FG plates using trigonometric shear deformation theory

Vagelis Plevris^{*1}, Lazreg Hadji^{2a} and Royal Madan^{3b}

¹Department of Civil and Environmental Engineering, Qatar University, P.O. Box 2713, Doha, Qatar

²Department of Civil Engineering, University of Tiaret, BP 78 Zaaroura, Tiaret, 14000, Algeria

³Department of Mechanical Engineering, Graphic Era (Deemed to be University), Dehradun 248002, India

(Received April 7, 2024, Revised October 17, 2024, Accepted October 18, 2024)

Abstract. This study investigates the free vibration behavior of functionally graded (FG) plates using trigonometric shear deformation plate theory. The novelty of this work lies in the incorporation of porosities, which are inherent in FG materials due to manufacturing processes, and their detailed impact on the vibrational performance of these plates. Unlike existing studies, this research comprehensively examines multiple porosity distribution patterns, including homogeneous, “O”, “X”, and “V” configurations, which are seldom analyzed together. The governing equations of motion are derived using Hamilton’s principle and solved analytically with the Navier method for simply supported boundary conditions. A key contribution of this study is the exploration of how porosity levels, distribution types, and geometry parameters collectively influence the natural frequencies of FG plates. The results highlight the significant effect of different porosity patterns, with “X”-shaped porosity yielding the highest natural frequency and homogeneous distribution leading to the lowest. Furthermore, the findings reveal that increased porosity levels can either enhance or diminish the vibrational characteristics depending on the distribution pattern. These insights provide valuable guidance for optimizing the design of FG plates for various engineering applications, such as aerospace and biomedical industries.

Keywords: FG plate; free vibration; functionally graded (FG) materials; porosity; trigonometric shear deformation theory

1. Introduction

Recent advancements in materials engineering have marked remarkable progress, with one of the most intriguing and innovative developments being the emergence of functionally graded materials (FGMs). Departing from conventional material design paradigms, FGMs have garnered considerable interest due to their unique and versatile properties. Unlike homogeneous materials, FGMs exhibit a continuous and seamless variation in their thermo-mechanical characteristics, making them distinct and highly adaptable.

FGMs represent a significant breakthrough in engineering and science, offering innovative solutions to multifaceted challenges encountered across various industries, including aerospace and biomedical sectors. Introduced by (Niknam *et al.* 2014), the concept of FGMs has sparked a wave of advancements in materials science and engineering. Fundamentally, FGMs are designed to possess tailored properties that smoothly transition from one end to another (Hadji *et al.* 2024b). Unlike traditional

materials with uniform characteristics, FGMs exhibit a gradient wherein properties such as mechanical strength, thermal conductivity, or chemical composition gradually vary in a predetermined direction. This unique feature holds tremendous potential for revolutionizing the design and performance of numerous components and systems.

The versatility of FGMs extends across diverse fields. In aerospace applications, they bolster the performance and resilience of vital components like turbine blades and heat shields. Within biomedical engineering, FGMs are tailored to mimic the mechanical properties of human tissues, leading to enhanced implants and prosthetics. Moreover, these materials exhibit promise in energy storage, electronic devices, and architectural contexts, where they optimize heat distribution and ensure structural robustness. Given the ubiquitous use of plates across sectors like civil engineering, marine, and aerospace engineering (Shahsavari *et al.* 2018), understanding their dynamic response, particularly in free vibration scenarios, is imperative.

Furthermore, it is notable that porosities may arise during the sintering phase in the production of FGMs, primarily due to the disparate solidification temperatures of their components (Zhu *et al.* 2001). In their investigation, (Wattanasakulpong *et al.* 2012) delved into the occurrence of porosities in FGM samples crafted through a sequential infiltration technique involving multiple steps. It is crucial to take into account the impact of porosity when designing structures made of FGMs under dynamic loading conditions. Recent research by (Wattanasakulpong and Ungbhakorn 2014) explored the vibration phenomena in FG

*Corresponding author, Associate Professor

E-mail: vplevris@qu.edu.qa

^aProfessor

E-mail: lazreg.hadji@univ-tiaret.dz

^bPh.D.

E-mail: royalmadan6293@gmail.com

beams containing porosities. Moreover, (Hadji *et al.* 2023a, Hadji *et al.* 2023b) explored the free vibration of porous FG beams by employing a higher-order shear deformation model. Their investigation delved into the influence of porosity, volume fraction, composition, and geometry on bending behavior and natural frequencies. Additionally, they employed hyperbolic shear deformation theory without shear correction factors to derive equations of motion via Hamilton's principle. Their primary objective was to examine how void distribution influences the free vibration characteristics of FGM beams using an advanced shear deformation model. (Noori *et al.* 2021) conducted dynamic analysis of FG porous beams using complementary functions method in the Laplace domain. Their parametric studies showed the influence of porosity constants, symmetric and asymmetric porosity distributions and damping ratios on the dynamic response.

In the same way, (Atmane *et al.* 2015) investigated a computational shear displacement model for vibrational analysis of functionally graded (FG) beams with porosities. (Ramteke *et al.* 2019) studied the effect of grading pattern and porosity on the eigen characteristics of porous FG structures. (Xu *et al.* 2021) developed wave propagation analysis of porous FG curved beams in thermal environment. (Meksi *et al.* 2023) analyzed the effect of temperature-dependent properties on the dynamic behavior of FG porous beams rested on variable elastic foundation. (Turan *et al.* 2023) conducted free vibration and buckling analysis of FG porous beams using a combination of analytical methods, finite element analysis, and artificial neural networks. Similarly, (Yaylaci *et al.* 2023) performed numerical investigations on the vibration and buckling behavior of FGM beams with edge cracks, employing the finite element method (FEM) and multilayer perceptron models. (Lakhdar *et al.* 2024) investigated the static bending and free vibration behavior of porous bi-directional FG doubly curved sandwich shells using a combination of third-order shear deformation theory and the p -version FEM.

FG plates have been thoroughly examined in the literature, particularly under static loading conditions (Doori *et al.* 2024). The free vibration of plates has been studied using various methods such as the finite element method (Eratli and Akoz 2002), differential quadrature method (Wu 2006), Vlasov model (Ayvaz and Oguzhan 2008), Integrated Force Method (Dhananjaya *et al.* 2010), meshfree model (Bui and Nguyen 2011). Two main approaches are typically employed when considering variation in FGMs. In the first approach, material property grading, such as power law or exponential law, is directly considered etc. (Abdalla *et al.* 2020, Nie and Batra 2010). In the second approach, variation in material composition, such as volume fraction, is manipulated (Arefi *et al.* 2020, Zenkour 2005). Due to differing thermo-mechanical properties, there is a risk of delamination of materials (Suresh and Mortensen 1997). So, a continuous gradation of materials is mathematically represented by assuming specific laws (Hadji *et al.* 2024a). Moreover, the material properties for these assumed FGMs are often estimated using methods such as the rule of mixture (ROM), Mori-

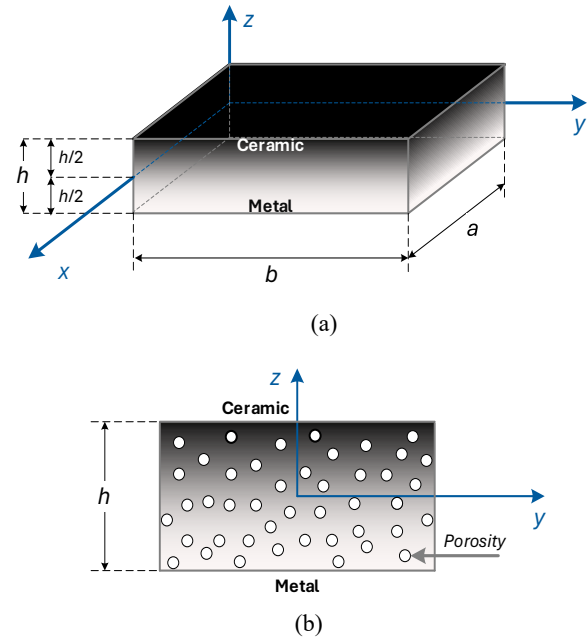


Fig. 1 Rectangular FG plate geometry and coordinate axes: (a) 3D view, (b) Cross section

Tanaka, and others (Benveniste 1987, Bruck and Rabin 1999).

As far as the authors are aware, there hasn't been a thorough investigation employing both shear and normal deformation theory to analyze the vibration characteristics of FG plates with porosities. Therefore, this study aims to explore the vibration response of porous FG plates, using trigonometric shear deformation plate theory to analyze their behavior. The material properties and porosity distribution within the plates change across the thickness direction according to the chosen pattern. The governing equations stem from the principle of least action, furnishing an analytical solution for evaluating the free vibration of porous FG plates. Furthermore, the analysis investigates how porosity distribution patterns and plate geometric factors, such as aspect ratio, affect the natural frequencies.

2. FG plates with porosities

Fig. 1 presents a schematic of an FG plate composed of two materials: ceramic and metal. The properties of the FG plate vary based on a power-law gradation, as described in Eq. (1) (Thai *et al.* 2014)

$$P(z) = (-P_b + P_t) \left(\frac{z}{h} + \frac{1}{2} \right)^k + P_b \quad (1)$$

In the above, the parameters P_b and P_t are values of the material properties at the bottom of the plate and the top of it, respectively, with k being the power-law exponent. The bottom surface of the FG plate is entirely composed of metal, whereas the top surface is solely ceramic, as shown in the figure. Therefore, varying the parameter k yields distinct metal-ceramic compositions throughout the plate.

When dealing with a porous FG plate, the rule of

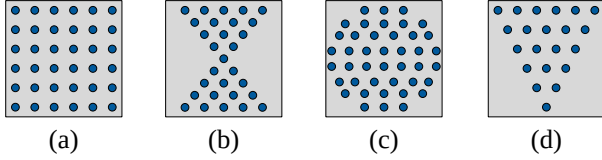


Fig. 2 Distributions of porosity: (a) “U” (Uniform), (b) “X” Shape, (c) “O” Shape, (d) “V” Shape

mixture necessitates adjustment. Specifically, for both uniform and non-uniform types of porosity distribution shapes, the revised rule of mixture is represented by Eqs. (2), (3), (4), and (5) for the Uniform, “X” shape, “O” shape, and “V” shape distributions, respectively (Hadj *et al.* 2019). Fig. 2 shows the shapes that correspond to these four distributions of porosity.

$$P(z) = (-P_b + P_t) \left(\frac{z}{h} + \frac{1}{2} \right)^k + P_b - \frac{\beta}{2} \cdot (P_b + P_t) \quad (2)$$

$$P(z) = (-P_b + P_t) \left(\frac{z}{h} + \frac{1}{2} \right)^k + P_b - \frac{\beta}{2} \cdot (P_b + P_t) \cdot \left(2 \frac{z}{h} \right) \quad (3)$$

$$P(z) = (-P_b + P_t) \left(\frac{z}{h} + \frac{1}{2} \right)^k + P_b - \frac{\beta}{2} \cdot (P_b + P_t) \cdot \left(1 - 2 \frac{|z|}{h} \right) \quad (4)$$

$$P(z) = (-P_b + P_t) \left(\frac{z}{h} + \frac{1}{2} \right)^k + P_b - \frac{\beta}{2} \cdot (P_b + P_t) \cdot \left(\frac{1}{2} + \frac{z}{h} \right) \quad (5)$$

Within these equations, $\beta < 0$ represents the porosity volume fraction. The properties of an ideal FG plate can be derived by taking the parameter β equal to zero.

2.1 Kinematic, strain and stress relations

The assumed field of displacements can be defined by the subsequent equations (Hadj *et al.* 2011)

$$\begin{aligned} U(x, y, z, t) &= u_0(x, y, t) - z \frac{\partial w_b}{\partial x} - f(z) \frac{\partial w_s}{\partial x} \\ V(x, y, z, t) &= v_0(x, y, t) - z \frac{\partial w_b}{\partial y} - f(z) \frac{\partial w_s}{\partial y} \end{aligned} \quad (6)$$

$$W(x, y, z, t) = w_b(x, y, t) + w_s(x, y, t)$$

Also, $f(z)$ refers to the variation of the shear strain in the transverse direction throughout the thickness of the plate. For this work, it has been adopted as (Khelifa *et al.* 2018)

$$f(z) = z - \frac{h}{\pi} \cdot \sin\left(\frac{\pi z}{h}\right) \quad (7)$$

The non-zero strains in Eq. (6) are

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + z k_x^b + f(z) k_x^s \\ \varepsilon_y &= \varepsilon_y^0 + z k_y^b + f(z) k_y^s \\ \gamma_{yz} &= g(z) \gamma_{yz}^s \\ \gamma_{xz} &= g(z) \gamma_{xz}^s \\ \varepsilon_z &= 0 \end{aligned} \quad (8)$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, \quad k_x^s = -\frac{\partial^2 w_s}{\partial x^2},$$

$$\begin{aligned} \varepsilon_y^0 &= \frac{\partial v_0}{\partial y}, \quad k_y^b = -\frac{\partial^2 w_b}{\partial y^2}, \quad k_y^s = -\frac{\partial^2 w_s}{\partial y^2}, \\ \gamma_{yz}^s &= \frac{\partial w_s}{\partial x}, \quad \gamma_{xz}^s = \frac{\partial w_s}{\partial y}, \quad g(z) = -f'(z) + 1 \end{aligned} \quad (9)$$

The constitutive relationships can be given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \quad (10)$$

where

$$\begin{aligned} C_{11} &= C_{22} = \frac{E(z)}{1-\nu^2} \\ C_{12} &= \frac{\nu E(z)}{1-\nu^2} \\ C_{44} &= C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)} \end{aligned} \quad (11)$$

2.2 Motion equations

In this context, the Principle of Least Action is applied to derive the equations of motion, and it is analytically formulated as follows (Reddy 2017)

$$\int_0^t (\delta U - \delta T) dt = 0 \quad (12)$$

In the above, δU denotes the variation of strain energy, while the one of the kinetic energy is denoted as δT and the one of the strain energy can be obtained by

$$\begin{aligned} \delta U &= \int_V [\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz}] dV \\ &= \int_A [N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_{xy} \delta \gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \\ &\quad + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta \gamma_{yz}^s + S_{xz}^s \delta \gamma_{xz}^s] dA = 0 \end{aligned} \quad (13)$$

In the above (Eq. (13)), A denotes the top surface of the plate. The stress resultants (denoted as N , M , and S) can be obtained as follows

$$\begin{aligned} \begin{Bmatrix} N_x & N_y & N_{xy} \\ M_x^b & M_y^b & M_{xy}^b \\ M_x^s & M_y^s & M_{xy}^s \end{Bmatrix} &= \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \\ (S_{xz}^s, S_{yz}^s) &= \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) g(z) dz \end{aligned} \quad (14)$$

If we combine Eqs. (10), (14) and we integrate across the plate's thickness, the parameters N , M , and S can be calculated as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \quad S = A^s \gamma \quad (15)$$

where

$$\begin{aligned} N &= \{N_x, N_y, N_{xy}\}^t, \quad M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t, \\ M^s &= \{M_x^s, M_y^s, M_{xy}^s\}^t \end{aligned}$$

$$\begin{aligned} \varepsilon &= \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}^t, \quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}^t, \\ k^s &= \{k_x^s, k_y^s, k_{xy}^s\}^t \end{aligned}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix},$$

$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix},$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix},$$

$$H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix},$$

$$S = \{S_{xz}^s, S_{yz}^s\}^t, \quad \gamma = \{\gamma_{xz}, \gamma_{yz}\}^t, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \quad (16)$$

The components of stiffness can be calculated as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}(1, z, z^2, f(z), z f(z), f^2(z)) \left\{ \frac{1}{2} \right\} dz$$

$$(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s) = (A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s)$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}[g(z)]^2 dz \quad (17)$$

The kinetic energy variation is

$$\delta T = \int_{-h/2}^h \int_A [\dot{u} \delta \dot{u} + \dot{v} \delta \dot{v} + \dot{w} \delta \dot{w}] \rho(z) dA dz$$

$$= \int_A \{ I_0 [\dot{u}_0 \delta \dot{u}_0 + \dot{v}_0 \delta \dot{v}_0 + (\dot{w}_b + \dot{w}_s)(\delta \dot{w}_b + \delta \dot{w}_s)]$$

$$- I_1 \left(\dot{u}_0 \frac{\partial \delta \dot{w}_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial x} \delta \dot{u}_0 + \dot{v}_0 \frac{\partial \delta \dot{w}_b}{\partial y} + \frac{\partial \dot{w}_b}{\partial y} \delta \dot{v}_0 \right)$$

$$- I_2 \left(\dot{u}_0 \frac{\partial \delta \dot{w}_s}{\partial x} + \frac{\partial \dot{w}_s}{\partial x} \delta \dot{u}_0 + \dot{v}_0 \frac{\partial \delta \dot{w}_s}{\partial y} + \frac{\partial \dot{w}_s}{\partial y} \delta \dot{v}_0 \right)$$

$$+ J_1 \left(\frac{\partial \dot{w}_b}{\partial x} \frac{\partial \delta \dot{w}_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial y} \frac{\partial \delta \dot{w}_b}{\partial y} \right) + K_2 \left(\frac{\partial \dot{w}_s}{\partial x} \frac{\partial \delta \dot{w}_s}{\partial x} + \frac{\partial \dot{w}_s}{\partial y} \frac{\partial \delta \dot{w}_s}{\partial y} \right)$$

$$+ J_2 \left(\frac{\partial \dot{w}_b}{\partial x} \frac{\partial \delta \dot{w}_s}{\partial x} + \frac{\partial \dot{w}_s}{\partial x} \frac{\partial \delta \dot{w}_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial y} \frac{\partial \delta \dot{w}_s}{\partial y} + \frac{\partial \dot{w}_s}{\partial y} \frac{\partial \delta \dot{w}_b}{\partial y} \right) \} dA \quad (18)$$

The notation with a dot-superscript means differentiation with respect to time t , while $\rho(z)$ represents the density. The parameters $I_0, I_1, I_2, J_1, J_2, K_2$ denote the mass inertias and they are given by

$$(I_0, I_1, I_2, J_1, J_2, K_2) = \int_{-h/2}^{h/2} (1, z, z^2, z f, z f^2) dz \quad (19)$$

By replacing the formulas for δU and δT from Eqs. (13) and (18) into Eq. (12), performing integration by parts, and subsequently gathering the coefficients, we can derive the motion equations as follows

$$\delta u_0: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} - J_1 \frac{\partial \ddot{w}_s}{\partial x}$$

$$\delta v_0: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} - J_1 \frac{\partial \ddot{w}_s}{\partial y}$$

$$\delta w_b: \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} =$$

$$I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_2 \nabla^2 \ddot{w}_b - J_2 \nabla^2 \ddot{w}_s$$

$$\delta w_s: \frac{\partial^2 M_x^s}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} =$$

$$I_0 (\ddot{w}_b + \ddot{w}_s) + J_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - J_2 \nabla^2 \ddot{w}_b - K_2 \nabla^2 \ddot{w}_s \quad (20)$$

The motion equations can be appropriately represented as

$$A_{11} \frac{\partial^2 u_0}{\partial x^2} + A_{66} \frac{\partial^2 u_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 v_0}{\partial x \partial y} - B_{11} \frac{\partial^3 w_b}{\partial x^3} -$$

$$(B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x \partial y^2} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 w_s}{\partial x \partial y^2} =$$

$$I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} - J_1 \frac{\partial \ddot{w}_s}{\partial x},$$

$$(A_{12} + A_{66}) \frac{\partial^2 u_0}{\partial x \partial y} + A_{66} \frac{\partial^2 v_0}{\partial x^2} + A_{22} \frac{\partial^2 v_0}{\partial y^2} - (B_{12} +$$

$$2B_{66}) \frac{\partial^3 w_b}{\partial x^2 \partial y} - B_{22} \frac{\partial^3 w_b}{\partial y^3} - B_{22}^s \frac{\partial^3 w_s}{\partial y^3} - (B_{12}^s +$$

$$2B_{66}^s) \frac{\partial^3 w_s}{\partial x^2 \partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} - J_1 \frac{\partial \ddot{w}_s}{\partial y},$$

$$B_{11} \frac{\partial^3 u_0}{\partial x^3} + (B_{12} + 2B_{66}) \frac{\partial^3 u_0}{\partial x \partial y^2} + (B_{12} + 2B_{66}) \frac{\partial^3 v_0}{\partial x^2 \partial y} +$$

$$B_{22} \frac{\partial^3 v_0}{\partial y^3} - D_{11} \frac{\partial^4 w_b}{\partial x^4} - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} -$$

$$D_{22} \frac{\partial^4 w_b}{\partial y^4} - D_{11}^s \frac{\partial^4 w_s}{\partial x^4} - 2(D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_s}{\partial x^2 \partial y^2} -$$

$$D_{22}^s \frac{\partial^4 w_s}{\partial y^4} = I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_2 \nabla^2 \ddot{w}_b -$$

$$J_2 \nabla^2 \ddot{w}_s,$$

$$B_{11}^s \frac{\partial^3 u}{\partial x^3} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 u}{\partial x \partial y^2} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 v}{\partial x^2 \partial y} +$$

$$B_{22}^s \frac{\partial^3 v}{\partial y^3} - D_{11}^s \frac{\partial^4 w_b}{\partial x^4} - 2(D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} -$$

$$D_{22}^s \frac{\partial^4 w_b}{\partial y^4} - H_{11}^s \frac{\partial^4 w_s}{\partial x^4} - 2(H_{12}^s + 2H_{66}^s) \frac{\partial^4 w_s}{\partial x^2 \partial y^2} -$$

$$H_{22}^s \frac{\partial^4 w_s}{\partial y^4} + A_{55}^s \frac{\partial^2 w_s}{\partial x^2} + A_{44}^s \frac{\partial^2 w_s}{\partial y^2} =$$

$$I_0 (\ddot{w}_b + \ddot{w}_s) + J_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - J_2 \nabla^2 \ddot{w}_b - K_2 \nabla^2 \ddot{w}_s \quad (21)$$

2.3 Navier solution

The Navier solution entails computing analytical solutions by representing displacement variables as the product of arbitrary parameters and established trigonometric functions. In our case, we focus on simply supported plates. As a result, displacements and moments are zero on the boundary. Specifically, the following boundary conditions are imposed at the side edges

$$v_0(0, y) = w_b(0, y) = w_s(0, y) = \frac{\partial w_b}{\partial y}(0, y) =$$

$$\frac{\partial w_s}{\partial y}(0, y) = 0$$

$$v_0(a, y) = w_b(a, y) = w_s(a, y) = \frac{\partial w_b}{\partial y}(a, y) =$$

$$\frac{\partial w_s}{\partial y}(a, y) = 0$$

$$N_x(0, y) = M_x^b(0, y) = M_x^s(0, y) = N_x(a, y) =$$

$$M_x^b(a, y) = M_x^s(a, y) = 0$$

$$u_0(x, 0) = w_b(x, 0) = w_s(x, 0) = \frac{\partial w_b}{\partial x}(x, 0) =$$

$$\frac{\partial w_s}{\partial x}(x, 0) = 0$$

$$u_0(x, b) = w_b(x, b) = w_s(x, b) = \frac{\partial w_b}{\partial x}(x, b) =$$

$$\frac{\partial w_s}{\partial x}(x, b) = 0$$

$$N_y(x, 0) = M_y^b(x, 0) = M_y^s(x, 0) = N_y(x, b) =$$

$$M_y^b(x, b) = M_y^s(x, b) = 0 \quad (22)$$

The displacement functions that satisfy the boundary conditions of Eq. (22) are expressed using the following

Table 1 Material properties for the two parts (Thai *et al.* 2014)

Type	Material	Modulus of Elasticity (GPa)	Density (kg/m ³)	Poisson's ratio
Metal	Al - Aluminum	70	2702	0.3
Ceramic	Al ₂ O ₃ - Aluminum oxide	380	3800	0.3

Fourier series, as outlined by (El Meiche *et al.* 2011)

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\lambda x) \sin(\mu y) e^{i\omega t} \\ V_{mn} \sin(\lambda x) \cos(\mu y) e^{i\omega t} \\ W_{bmn} \sin(\lambda x) \sin(\mu y) e^{i\omega t} \\ W_{smn} \sin(\lambda x) \sin(\mu y) e^{i\omega t} \end{Bmatrix} \quad (23)$$

In the given equation, U_{mn} , V_{mn} , W_{bmn} , and W_{smn} , represent some arbitrary parameters, while $\omega = \omega_{mn}$ is the eigenfrequency which corresponds to the (m, n) -th eigenmode. Upon substituting the expression of Eq. (23) into the set of Eq. (21), and performing integration over the domain of the solution, subsequent mathematical manipulations lead to

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{34} & m_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (24)$$

In which

$$\begin{aligned} S_{11} &= A_{11}\lambda^2 + A_{66}\mu^2, \\ S_{12} &= \lambda \cdot \mu (A_{12} + A_{66}), \\ S_{13} &= -\lambda (B_{11}\lambda^2 + (B_{12} + 2B_{66})\mu^2), \\ S_{14} &= -\lambda (B_{11}^s\lambda^2 + (B_{12}^s + 2B_{66}^s)\mu^2), \\ S_{22} &= A_{66}\lambda^2 + A_{22}\mu^2, \\ S_{23} &= -\mu ((B_{12} + 2B_{66})\lambda^2 + B_{22}\mu^2), \\ S_{24} &= -\mu ((B_{12}^s + 2B_{66}^s)\lambda^2 + B_{22}^s\mu^2), \\ S_{33} &= D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4, \\ S_{34} &= D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\mu^2 + D_{22}^s\mu^4, \\ S_{44} &= H_{11}^s\lambda^4 + 2(H_{12}^s + 2H_{66}^s)\lambda^2\mu^2 + H_{22}^s\mu^4 + A_{55}\lambda^2 + A_{44}\mu^2, \\ m_{11} &= m_{22} = I_1, \\ m_{33} &= I_0 + I_2(\lambda^2 + \mu^2), \\ m_{34} &= I_0 + J_2(\lambda^2 + \mu^2), \\ m_{44} &= I_0 + K_2(\lambda^2 + \mu^2) \end{aligned} \quad (25)$$

3. Numerical analysis and results

This section presents the numerical results on the free vibration behavior of FG plates, focusing on the effects of porosity distribution patterns and geometric properties. The FG plates are composed of a metal component, Aluminum (Al), and a ceramic component, Aluminum oxide (Al₂O₃). Table 1 details the material properties used in this investigation.

Multiple numerical results are used to validate the proposed approach, confirm the accuracy of the current solution and explore the effects of the power-law index,

Table 2 Nondimensional natural frequency ($\bar{\omega}$) of Al/Al₂O₃ FG plates ($a/b=0.5$)

a/h	Mode (m,n)	Power-law index					
		0	0.5	1	2	5	10
5	Present	3.4416	2.9349	2.6478	2.3948	2.2259	2.1403
	(1,1) (Nguyen 2015)	3.4464	2.9380	2.6509	2.3971	2.2260	2.1432
	Present	5.2822	4.5187	4.0787	3.6804	3.3914	3.2506
	(1,2) (Nguyen 2015)	5.2932	4.5258	4.0860	3.6859	3.3919	3.2574
	Present	11.5614	9.9754	9.0201	8.0915	7.2869	6.9259
	(2,2) (Nguyen 2015)	11.6113	10.0109	9.0538	8.1181	7.2951	6.9568
20	Present	3.7123	3.1458	2.8353	2.5771	2.4401	2.3618
	(1,1) (Nguyen 2015)	3.7127	3.1455	2.8355	2.5773	2.4401	2.3622
	Present	5.9199	5.0179	4.5228	4.1100	3.8881	3.7621
	(1,2) (Nguyen 2015)	5.9209	5.0176	4.5234	4.1104	3.8880	3.7629
	Present	14.6075	12.3964	11.1749	10.1456	9.5648	9.2434
	(2,2) (Nguyen 2015)	14.6131	12.3983	11.1785	10.1482	9.5645	9.2471

Table 3 Nondimensional natural frequency ($\bar{\omega}$) of Al/Al₂O₃ FG plates ($a/b=1$)

a/h	Mode (m,n)	Power-law index					
		0	0.5	1	2	5	10
5	Present	5.2822	4.5187	4.0787	3.6804	3.3914	3.2506
	(1,1) (Nguyen 2015)	5.2932	4.5258	4.0860	3.6859	3.3919	3.2574
	Present	11.5614	9.9754	9.0201	8.0915	7.2869	6.9259
	(1,2) (Nguyen 2015)	11.6113	10.0109	9.0538	8.1181	7.2951	6.9568
	Present	16.7328	14.5155	13.1438	11.7527	10.4400	9.8711
	(2,2) (Nguyen 2015)	16.8351	14.5888	13.2140	11.8101	10.4647	9.9360
20	Present	5.9199	5.0179	4.5228	4.1100	3.8881	3.7621
	(1,1) (Nguyen 2015)	5.9209	5.0176	4.5234	4.1104	3.8880	3.7629
	Present	14.6075	12.3964	11.1749	10.1456	9.5648	9.2434
	(1,2) (Nguyen 2015)	14.6131	12.3983	11.1785	10.1482	9.5645	9.2471
	Present	23.0786	19.6066	17.6775	16.0355	15.0691	14.5459
	(2,2) (Nguyen 2015)	23.0925	19.6126	17.6865	16.0419	15.0685	14.5550

porosity distribution pattern, length-to-thickness ratio, and porosity parameters on the natural frequency of FG porous plates. For ease of analysis, the unitless parameter $\bar{\omega}$ is introduced as shown in Eq. (26), which expresses a non-dimensional natural frequency (Nguyen 2015)

$$\bar{\omega} = \frac{\omega \cdot a^2}{h} \sqrt{\frac{\rho_c}{E_c}} \quad (26)$$

Tables 2 and 3 depict the relationship between frequency and the grading parameter (k) for a FG plate. Maintaining an aspect ratio (a/h) of 0.5 and varying the “length to height” (L/h) ratio from 5 to 20 yields an escalation in natural frequency. Moreover, increasing the aspect ratio (a/h) correlates with a corresponding

Table 4 Effects of the shape of the distribution of porosity, on the frequency (ω) of FG simply supported $\text{Al}/\text{Al}_2\text{O}_3$ plates, $k=1$

a/h	Theory	β	Distribution shape of the porosity			
			"U" (Uniform)	"X"	"O"	"V"
5	CPT	0	4.4048	4.4048	4.4048	4.4048
		0.1	4.3040	4.5137	4.4240	4.4222
		0.2	4.1595	4.6043	4.4425	4.4410
	Present	0	4.0784	4.0784	4.0784	4.0784
		0.1	3.9984	4.1644	4.0896	4.0924
		0.2	3.8823	4.2351	4.0994	4.1076
10	CPT	0	4.5194	4.5194	4.5194	4.5194
		0.1	4.4179	4.6319	4.5428	4.5388
		0.2	4.2723	4.7251	4.5659	4.5597
	Present	0	4.4193	4.4193	4.4193	4.4193
		0.1	4.3244	4.5244	4.4397	4.4374
		0.2	4.1875	4.6111	4.4595	4.4569
20	CPT	0	4.5493	4.5493	4.5493	4.5493
		0.1	4.4477	4.6629	4.5738	4.5692
		0.2	4.3018	4.7567	4.5982	4.5907
	Present	0	4.5228	4.5228	4.5228	4.5228
		0.1	4.4229	4.6344	4.5465	4.5424
		0.2	4.2793	4.7264	4.5699	4.5634

augmentation in the frequency. The power-law index signifies the change in metal-ceramic composition within the FG plate. Elevating the parameter k leads to a reduction in the plate's frequency. This trend arises because higher values of k indicate a metal-rich plate, implying lower ceramic content and higher metal content. Metals typically exhibit lower stiffness and higher flexibility compared to ceramics, which are characterized by higher stiffness. Although density also influences frequency tuning, in the present context, the materials utilized manifest a more pronounced difference in modulus than in density. Consequently, the upsurge in metal content with increasing values of k culminates in a decline in frequency.

In FGMs, modifications to material properties are achieved through adjustments in microstructure, composition of the reinforced material, and the introduction of variations in porosity within the structure (Barbaros *et al.* 2022). Manufacturing these materials can be accomplished through various techniques, including gas-based methods, liquid-based processes, as well as solid-phase methods (like powder metallurgy) (Madan and Bhowmick 2022, Zhou *et al.* 2021). Functionally graded plates have been successfully produced by numerous researchers using these methods, with additional details available in the provided references (Chen *et al.* 2019, El-Galy *et al.* 2019). Although these methods aim to produce high-density products, the presence of porosity in a structure can occur due to various factors, including incorrect operational procedures such as insufficient sintering temperature or time, improper solidification, and variations in particle shape or size (Rajan 2014, Watanabe *et al.* 2009).

Consequently, the influence of the porosity parameter on vibration frequency has been observed. Table 3 presents the frequency variation of a porous FG plate which is

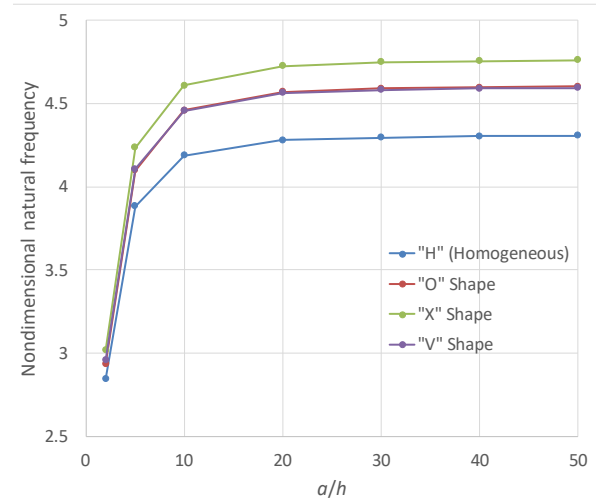


Fig. 3 The effect of the ratio a/h on the frequency of the plate ($k=1$ and $\beta=0.2$)

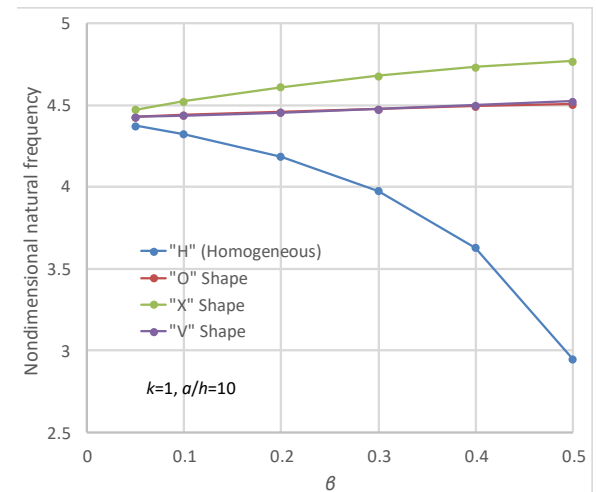


Fig. 4 Natural frequency of the FG plate vs Porosity coefficient β ($k=1$ and $a=10h$)

simply supported, for different values of aspect ratio (a/h), porosity parameter (β), and porosity distribution patterns. Additionally, results from classical plate theory (CPT) are included for comparative analysis. The findings show that an increase in porosity is associated with a decrease in the natural frequency. Moreover, as the plate thickness increases, the discrepancy between CPT and the current solution becomes more pronounced. However, there is noteworthy agreement between CPT and the current solution results, especially for thin plates. It is important to highlight that the CPT results are derived by setting $f(z)=0$ in Eq. (6).

Fig. 3 depicts the impact of the a/h ratio on the frequency of the functionally graded with different distribution patterns for porosity. This ratio corresponds to the length (a) to thickness (h) ratio. It is shown that the "X"-shaped porosity type yields the largest natural frequency, while plates with homogeneous (uniform) porosity distributions exhibit the lowest natural frequency. This underscores the significant influence of pore arrangement and shape on the material's vibrational

behavior, with the “X”-shaped configuration resulting in superior natural frequency compared to uniform distributions.

Fig. 4 shows how the porosity coefficient (β) affects the non-dimensional fundamental frequency for various porosity shapes. As β increases from 0.1 to 0.5, the natural frequency rises for “O”, “X”, and “V” types. Conversely, a reverse trend is observed for homogeneous shapes, leading to a significant decrease in frequency. Notably, for “O” and “V” shapes, frequencies are nearly identical, except for the case $\beta=0.5$, where a slightly higher frequency is observed for the “V” type compared to the “O” type. Upon comparison, the natural frequency is highest for the “X” type, followed by “O”/“V” types, and then homogeneous shapes. Homogeneous porosity reduces the structure’s stiffness, resulting in a lower natural frequency. Conversely, in other porosity shapes, increased porosity introduces a damping effect on the plate.

4. Conclusions

This research offers a comprehensive understanding of the complex relationship between porosity distribution patterns and the vibrational behavior of FG porous plates, using trigonometric shear deformation theory. Through detailed analysis, four distinct porosity distribution types were examined, revealing their specific effects on the dynamic response of the plates.

The study explored the combined influence of several key factors, including porosity levels, distribution patterns, the grading parameter of the FG material, and geometric characteristics such as the length-to-thickness ratio. By systematically varying these factors, we gained valuable insights into their individual and combined impacts on the vibrational performance of the plates, providing a deeper understanding of FG plate dynamics.

To ensure the reliability and accuracy of our findings, we conducted thorough comparisons with data from the literature, which confirmed the validity of the analytical solution presented. This careful validation process strengthens our conclusions and enhances the credibility of the research outcomes.

Our investigation revealed intriguing insights into the relationship between porosity distribution patterns and natural frequency. Remarkably, the natural frequency was found to peak for the “X”-shaped porosity distribution, while reaching its nadir for homogeneous porosity distribution. Moreover, we observed a notable trend wherein increasing the porosity parameter led to enhanced natural frequencies for “X”, “O”, and “V” shape distributions, while conversely diminishing frequencies in the case of homogeneous porosity distribution.

In essence, our study not only elucidates the intricate dynamics of FG porous plates but also provides valuable insights for optimizing their vibrational characteristics. These findings hold significant implications for diverse engineering applications, from aerospace components to biomedical implants, where the vibrational behavior of FG plates plays a crucial role in performance and reliability.

Moving forward, further exploration into advanced manufacturing techniques and innovative design strategies could pave the way for even more tailored and optimized FG plate structures, enhancing their functionality and applicability across various domains.

Author contributions

Authors 1 and 2 conceptualized the project, initiated its implementation, and suggested numerical analysis examples. Authors 2 and 3 conducted the numerical analysis and evaluated the outcomes. Authors 1 and 3 worked on the development of the mathematical model and validated the theory. The document was collaboratively authored by all contributors. All authors participated in result discussions, manuscript review, and approval of the final manuscript.

Conflict of interest

The authors have revealed no possible conflicts of interest pertaining to the authorship, research and publication of the present manuscript.

Data availability statements

Data from the current study, whether generated or analyzed, can be obtained from the authors upon request.

References

- Abdalla, H.M.A., Casagrande, D. and Moro, L. (2020), “Thermo-mechanical analysis and optimization of functionally graded rotating disks”, *J. Strain Anal. Eng. Des.*, **55**(5-6), 159-171. <https://doi.org/10.1177/0309324720904793>.
- Arefi, M., Firouzeh, S., Mohammad-Rezaei Bidgoli, E. and Civalek, Ö. (2020), “Analysis of porous micro-plates reinforced with FG-GNPs based on Reddy plate theory”, *Compos. Struct.*, **247**, 112391. <https://doi.org/10.1016/j.compstruct.2020.112391>.
- Atmane, H.A., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), “A computational shear displacement model for vibrational analysis of functionally graded beams with porosities”, *Steel Compos. Struct.*, **19**(2), 369-385. <https://doi.org/10.12989/scs.2015.19.2.369>.
- Ayvaz, Y. and Oguzhan, C.B. (2008), “Free vibration analysis of plates resting on elastic foundations using modified Vlasov model”, *Struct. Eng. Mech.*, **28**(6), 635-658. <https://doi.org/10.12989/sem.2008.28.6.635>.
- Barbaros, I., Yang, Y., Safaei, B., Yang, Z., Qin, Z. and Asmael, M. (2022), “State-of-the-art review of fabrication, application, and mechanical properties of functionally graded porous nanocomposite materials”, *Nanotechnol. Rev.*, **11**(1), 321-371. <https://doi.org/10.1515/ntrev-2022-0017>.
- Benveniste, Y. (1987), “A new approach to the application of Mori-Tanaka’s theory in composite materials”, *Mech. Mater.*, **6**(2), 147-157. [https://doi.org/10.1016/0167-6636\(87\)90005-6](https://doi.org/10.1016/0167-6636(87)90005-6).
- Bruck, H.A. and Rabin, B.H. (1999), “Evaluation of rule-of-mixtures predictions of thermal expansion in powder-processed Ni–Al₂O₃ composites”, *J. Am. Ceram. Soc.*, **82**(10), 2927-

2930. <https://doi.org/10.1111/j.1151-2916.1999.tb02182.x>.
- Bui, T.Q. and Nguyen, M.N. (2011), "A novel meshfree model for buckling and vibration analysis of rectangular orthotropic plates", *Struct. Eng. Mech.*, **39**(4), 579-598. <https://doi.org/10.12989/sem.2011.39.4.579>.
- Chen, X., Liu, K., Guo, W., Gangil, N., Siddiquee, A.N. and Kononov, S. (2019), "The fabrication of NiTi shape memory alloy by selective laser melting: A review", *Rapid Prototyp. J.*, **25**(8), 1421-1432. <https://doi.org/10.1108/RPJ-11-2018-0292>.
- Dhananjaya, H.R., Nagabhushanam, J., Pandey, P.C. and Jumaat, M.Z. (2010), "New twelve node serendipity quadrilateral plate bending element based on Mindlin-Reissner theory using Integrated Force Method", *Struct. Eng. Mech.*, **36**(5), 625-642. <https://doi.org/10.12989/sem.2010.36.5.625>.
- Doori, S.G.M., Noori, A.R. and Etemadi, A. (2024), "Static response of functionally graded porous circular plates via finite element method", *Arab. J. Sci. Eng.*, **49**(10), 14167-14181. <https://doi.org/10.1007/s13369-024-08914-w>.
- El Meiche, N., Tounsi, A., Ziane, N., Mechab, I. and Adda.Bedia, E.A. (2011), "A new hyperbolic shear deformation theory for buckling and vibration of functionally graded sandwich plate", *Int. J. Mech. Sci.*, **53**(4), 237-247. <https://doi.org/10.1016/j.ijmecsci.2011.01.004>.
- El-Galy, I.M., Saleh, B.I. and Ahmed, M.H. (2019), "Functionally graded materials classifications and development trends from industrial point of view", *SN Appl. Sci.*, **1**(11), 1378. <https://doi.org/10.1007/s42452-019-1413-4>.
- Eratli, N. and Akoz, A.Y. (2002), "Free vibration analysis of Reissner plates by mixed finite element", *Struct. Eng. Mech.*, **13**(3), 277-298. <https://doi.org/10.12989/sem.2002.13.3.277>.
- Hadji, B., Rabia, B. and Daouadi, T.H. (2019), "Influence of the distribution shape of porosity on the bending FGM new plate model resting on elastic foundations", *Struct. Eng. Mech.*, **72**(1), 61-70. <https://doi.org/10.12989/sem.2019.72.1.061>.
- Hadji, L., Atmane, H.A., Tounsi, A., Mechab, I. and Adda Bedia, E.A. (2011), "Free vibration of functionally graded sandwich plates using four-variable refined plate theory", *Appl. Math. Mech.*, **32**(7), 925-942. <https://doi.org/10.1007/s10483-011-1470-9>.
- Hadji, L., Plevris, V. and Madan, R. (2023a), "Investigating the impact of porosity distribution and grading parameter on the free vibration of functionally graded beams", *9th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPdyn 2023)*, Athens, Greece, June.
- Hadji, L., Plevris, V. and Madan, R. (2023b), "A static and free vibration analysis of porous functionally graded beams", *2nd International Conference on Civil Infrastructure and Construction (CIC 2023)*, Doha, Qatar, February.
- Hadji, L., Plevris, V. and Papazafeiropoulos, G. (2024b), "Investigation of the static bending response of FGM sandwich plates", *J. Appl. Comput. Mech.*, **10**(1), 26-37. <https://doi.org/10.22055/jacm.2023.44278.4194>.
- Hadji, L., Plevris, V., Madan, R. and Ait Atmane, H. (2024a), "Multi-directional functionally graded sandwich plates: buckling and free vibration analysis with refined plate models under various boundary conditions", *Comput.*, **12**(4), 65. <https://doi.org/10.3390/computation12040065>.
- Khelifa, Z., Hadji, L., Daouadi, T.H. and Bourada, M. (2018), "Buckling response with stretching effect of carbon nanotube-reinforced composite beams resting on elastic foundation", *Struct. Eng. Mech.*, **67**(2), 125-130. <https://doi.org/10.12989/sem.2018.67.2.125>.
- Lakhdar, Z., Chorfi, S.M., Belalia, S.A., Khedher, K.M., Alluqmani, A.E., Tounsi, A. and Yaylacı, M. (2024), "Free vibration and bending analysis of porous bi-directional FGM sandwich shell using a TSDT p-version finite element method", *Acta Mechanica*, **235**(6), 3657-3686. <https://doi.org/10.1007/s00707-024-03909-y>.
- Madan, R. and Bhowmick, S. (2022), "Fabrication, microstructural characterization and finite element analysis of functionally graded Al-Al₂O₃ disk using powder metallurgy technique", *Mater. Today Commun.*, **32**, 103878. <https://doi.org/10.1016/j.mtcomm.2022.103878>.
- Meksi, A., Sekkal, M., Bouiadja, R.B., Benyoucef, S. and Tounsi, A. (2023), "Assessing the effect of temperature-dependent properties on the dynamic behavior of FG porous beams rested on variable elastic foundation", *Struct. Eng. Mech.*, **85**(6), 717-728. <https://doi.org/10.12989/sem.2023.85.6.717>.
- Nguyen, T.K. (2015), "A higher-order hyperbolic shear deformation plate model for analysis of functionally graded materials", *Int. J. Mech. Mater. Des.*, **11**(2), 203-219. <https://doi.org/10.1007/s10999-014-9260-3>.
- Nie, G.J. and Batra, R.C. (2010), "Stress analysis and material tailoring in isotropic linear thermoelastic incompressible functionally graded rotating disks of variable thickness", *Compos. Struct.*, **92**(3), 720-729. <https://doi.org/10.1016/j.compstruct.2009.08.052>.
- Niknam, H., Fallah, A. and Aghdam, M.M. (2014), "Nonlinear bending of functionally graded tapered beams subjected to thermal and mechanical loading", *Int. J. Nonlin. Mech.*, **65**, 141-147. <https://doi.org/10.1016/j.ijnonlinmec.2014.05.011>.
- Noori, A.R., Aslan, T.A. and Temel, B. (2021), "Dynamic analysis of functionally graded porous beams using complementary functions method in the Laplace domain", *Compos. Struct.*, **256**, 113094. <https://doi.org/10.1016/j.compstruct.2020.113094>.
- Rajan, T.P.D. and Pai, B.C. (2014), "Developments in processing of functionally gradient metals and metal-ceramic composites: A review", *Acta Metallurgica Sinica (English Lett.)*, **27**(5), 825-838. <https://doi.org/10.1007/s40195-014-0142-3>.
- Ramteke, P.M., Panda, S.K. and Sharma, N. (2019), "Effect of grading pattern and porosity on the eigen characteristics of porous functionally graded structure", *Steel Compos. Struct.*, **33**(6), 865-875. <https://doi.org/10.12989/scs.2019.33.6.865>.
- Reddy, J.N. (2017), *Energy Principles and Variational Methods in Applied Mechanics*, 3rd Edition, Wiley.
- Shahsavari, D., Shahsavari, M., Li, L. and Karami, B. (2018), "A novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation", *Aerosp. Sci. Technol.*, **72**, 134-149. <https://doi.org/10.1016/j.ast.2017.11.004>.
- Suresh, S. and Mortensen, A. (1997), "Functionally graded metals and metal-ceramic composites: Part 2 Thermomechanical behaviour", *Int. Mater. Rev.*, **42**(3), 85-116. <https://doi.org/10.1179/imr.1997.42.3.85>.
- Thai, H.T., Nguyen, T.K., Vo, T.P. and Lee, J. (2014), "Analysis of functionally graded sandwich plates using a new first-order shear deformation theory", *Eur. J. Mech.-A/Solid.*, **45**, 211-225. <https://doi.org/10.1016/j.euromechsol.2013.12.008>.
- Turan, M., Uzun Yaylacı, E. and Yaylacı, M. (2023), "Free vibration and buckling of functionally graded porous beams using analytical, finite element, and artificial neural network methods", *Arch. Appl. Mech.*, **93**(4), 1351-1372. <https://doi.org/10.1007/s00419-022-02332-w>.
- Watanabe, Y., Inaguma, Y., Sato, H. and Miura-Fujiwara, E. (2009), "A novel fabrication method for functionally graded materials under centrifugal force: The centrifugal mixed-powder method", *Mater.*, **2**(4), 2510-2525. <https://doi.org/10.3390/ma2042510>.
- Wattanasakulpong, N. and Ungbhakorn, V. (2014), "Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities", *Aerosp. Sci. Technol.*, **32**(1), 111-120. <https://doi.org/10.1016/j.ast.2013.12.002>.
- Wattanasakulpong, N., Gangadhara Prusty, B., Kelly, D.W. and

- Hoffman, M. (2012), "Free vibration analysis of layered functionally graded beams with experimental validation", *Mater. Des.*, **36**, 182-190. <https://doi.org/10.1016/j.matdes.2011.10.049>.
- Wu, L. (2006), "Thermal vibration analysis of thick laminated plates by the moving least squares differential quadrature method", *Struct. Eng. Mech.*, **22**(3), 331-349. <https://doi.org/10.12989/sem.2006.22.3.331>.
- Xu, X., Zhang, C., Musharavati, F., Sebaey, T.A. and Khan, A. (2021), "Wave propagation analysis of porous functionally graded curved beams in the thermal environment", *Struct. Eng. Mech.*, **79**(6), 665-675. <https://doi.org/10.12989/sem.2021.79.6.665>.
- Yaylaci, M., Yaylaci, E.U., Ozdemir, M.E., Ozturk, Ş. and Sesli, H. (2023), "Vibration and buckling analyses of FGM beam with edge crack: Finite element and multilayer perceptron methods", *Steel Compos. Struct.*, **46**(4), 565-575. <https://doi.org/10.12989/scs.2023.46.4.565>.
- Zenkour, A.M. (2005), "A comprehensive analysis of functionally graded sandwich plates: Part 2—Buckling and free vibration", *Int. J. Solid. Struct.*, **42**(18), 5243-5258. <https://doi.org/10.1016/j.ijsolstr.2005.02.016>.
- Zhou, H., Li, Y., Wang, H., Ran, M., Tong, Z., Zhang, W., Liu, J. and Zheng, W. (2021), "Fabrication of functionally graded diamond/Al composites by liquid–solid separation technology", *Mater.*, **14**(12), 3205. <https://doi.org/10.3390/ma14123205>.
- Zhu, J., Lai, Z., Yin, Z., Jeon, J. and Lee, S. (2001), "Fabrication of ZrO₂–NiCr functionally graded material by powder metallurgy", *Mater. Chem. Phys.*, **68**(1), 130-135. [https://doi.org/10.1016/S0254-0584\(00\)00355-2](https://doi.org/10.1016/S0254-0584(00)00355-2).